Rolls Royce four blade fan assembly: initial results

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October 1, 2008

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1 Summary and conclusions

Emulation techniques were used to analyse results of the fan assembly simulator. Initially, the fan assembly was run using a 2^4 factorial design and the results showed linear effects with minimal interactions.

The linear results were confirmed when adding 14 extra runs of another 2^4 design, together with an additional centre point at the origin. The results changed very little. Emulation techniques were used to give main effects plots, which were mainly linear.

The results of the full data set, with 31 runs are presented.

1.1 Future work

We suspect that the high linearity of the results may be due to the small experimentation range, and thus when extending the range, some curvature may well appear. More experiments will be performed with extended ranges.

A second stage of work is to be carried out with the eight blade assembly model. This will enable us to assess the results for the four blade assembly when considering a bigger assembly. It is hoped to complete this work in early December, 2008.

2 Experiment

Each experiment consists in adding small amounts of "mass" to the blades. The amount of mass can take positive or negative values and this creates each run. Then the assembly is tested for a range of different input frequencies, which gives amplitude and phase responses. The output is the frequency at which the maximum amplitude occurs.

The masses (factors) are labelled as $x_1, \ldots, x_4$. The outputs are maximum amplitude for each blade $a_1, \ldots, a_4$ and frequency $f_1, \ldots, f_4$ at which that maximum resonant frequency occurs.

2.1 Design

The following 31 runs were used:

- A full 2^4 design, with 16 runs was constructed. This design was built with all possible combinations of levels ± 1, where the levels correspond

to masses of $\pm 3 \times 10^{-6}$.

- A second design with 14 runs was constructed by first taking a full 2^4 factorial design with levels corresponding to masses of $\pm 1.5 \times 10^{-6}$, and then removing the runs $----$ and $++++$. The reason for removing this two runs is that balanced runs give the same response value for all the outputs.

- A center point was added.

For all the $31 = 16 + 14 + 1$ runs, damping was kept fixed to 0.01 and the frequency range was $(39, 46)$ in frequency steps of 0.05. In all the analyses performed, the design coordinates were linearly rescaled to lie in $[0, 1]^4$.

2.2 Results and analysis

Two outputs were analysed: amplitude a_1 and frequency f_1, i.e. responses on blade one only. The results presented correspond to the analysis of the full data set (31 runs). The data set is presented in Table 2 in Appendix A.

Due to the symmetry of the configuration, analysis of outputs at other blades will give similar results, up to a permutation of the factors, see details in Appendix B.

Response	$RMSE$ (% range)	Estimated factor parameters				
			x_1	x_2	x_3	x_4
a_1	1.2	θ_i	0.3003	0.2593	0.1298	0.2574
		p_i	1.999	1.999	1.999	1.999
f_1	4.1	θ_i	0.3180	0.3104	0.2613	0.2877
		p_i	1.8585	1.999	1.7333	1.9462

Table 1: Emulation results.

Emulators based on gaussian process with exponential covariance structure were fitted separately for responses a_1 and f_1, see Table 1. Crossvalidation diagnostics plots (true vs. predicted) in Figures 1 and 2 show a good emulator fit, with cross validation root mean square errors being 1.2% and 4.1% of the response range, respectively.

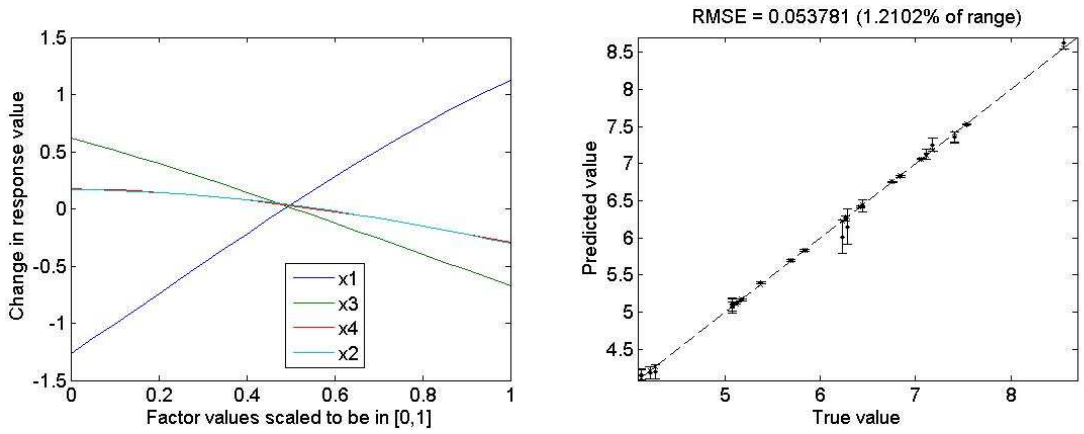

Figure 1: Amplitude a_1 emulation results: main effects plot (left) and cros-validation results plot (true vs. predicted values) (right plot).

2.2.1 Amplitude a_1

Amplitude was found linearly dependent on all the factors, with very little curvature involved, see main effects plot in Figure 1. The main factor is x_1, with positive correlation then x_3 and finally x_2 and x_4. In other words, amplitude on blade one increases linearly with mass added on blade one itself, and with mass substracted from blade three opposite. Masses of blades two and four have minimal effect.

The overall low values of length scale θ_i observed relate to factors being important to the model, while all the exponents p_i had value very close to two, which refers to the smoothness of the model fitted.

2.2.2 Frequency f_1

Frequency was also linearly dependent on all the factors, with slightly more curvature than observed for amplitude, see main effects plot in Figure 2. Here the factor ranking is x_3 as main factor then x_2 and x_2 and finally x_1. However, their effects are even, all of them negatively correlated with the frequency observed.

For this response, also low values of length scale θ_i were observed. The fit allowed for values of p_i lower than two, thus reflecting a less smooth response.

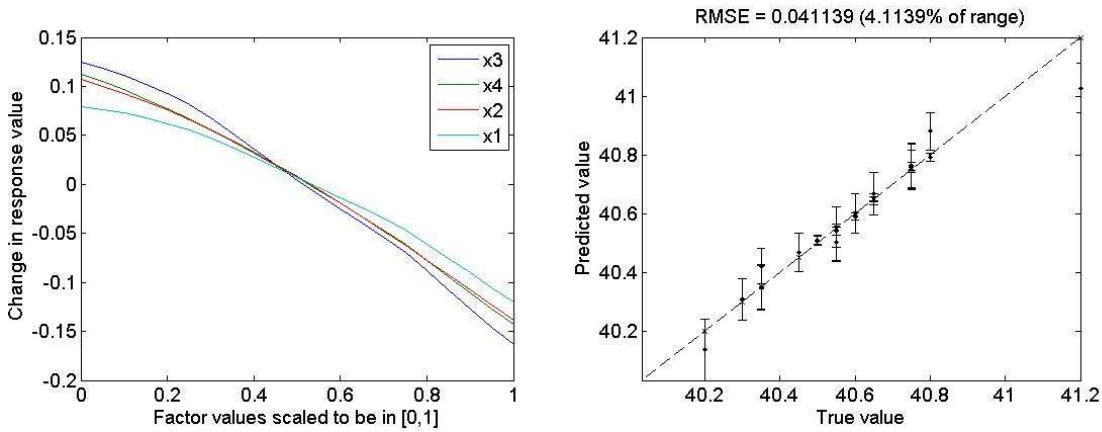

Figure 2: Frequency f_1 emulation results: main effects plot (left) and cross-validation true vs. predicted values plot (right).

Acknowledgments

The Rolls Royce four blade assembly model was used to generate data. The ANSYS code for that model was kindly provided by Hilmi Kurt-Elli (Rolls Royce). Hugo Maruri-Aguilar is also grateful for the training received at the Rolls Royce plant in Derby.

Appendices

A Data set

The experiment dataset is presented in Table 2. The factors are $x_1, \ldots, x_4$, while the outputs are $a_1, \ldots, a_4, f_1, \ldots, f_4$. The first 16 rows of the data set is the complete 2^4 design, then the central point and 14 rows of the second 2^4 design. See Section 2.1 for coding and further details.

x_1	x_2	x_3	x_4	a_1	a_2	a_3	a_4	f_1	f_2	f_3	f_4
−1	−1	−1	−1	6.23	6.23	6.23	6.23	41.2	41.2	41.2	41.2
−1	−1	−1	1	5.08	4.27	5.08	8.56	40.75	40.75	40.75	40.8
−1	−1	1	−1	4.27	5.08	8.56	5.08	40.75	40.75	40.8	40.75
−1	−1	1	1	4.12	4.12	7.41	7.41	40.45	40.45	40.55	40.55
−1	1	−1	−1	5.08	8.56	5.08	4.27	40.75	40.8	40.75	40.75
−1	1	−1	1	5.08	7.11	5.08	7.11	40.55	40.6	40.55	40.6
−1	1	1	−1	4.12	7.41	7.41	4.12	40.45	40.55	40.55	40.45
−1	1	1	1	4.21	6.44	7.18	6.44	40.3	40.35	40.35	40.35
1	−1	−1	−1	8.56	5.08	4.27	5.08	40.8	40.75	40.75	40.75
1	−1	−1	1	7.41	4.12	4.12	7.41	40.55	40.45	40.45	40.55
1	−1	1	−1	7.11	5.08	7.11	5.08	40.6	40.55	40.6	40.55
1	−1	1	1	6.44	4.21	6.44	7.18	40.35	40.3	40.35	40.35
1	1	−1	−1	7.41	7.41	4.12	4.12	40.55	40.55	40.45	40.45
1	1	−1	1	7.18	6.44	4.21	6.44	40.35	40.35	40.3	40.35
1	1	1	−1	6.44	7.18	6.44	4.21	40.35	40.35	40.35	40.3
1	1	1	1	6.29	6.29	6.29	6.29	40.2	40.2	40.2	40.2
0	0	0	0	6.26	6.26	6.26	6.26	40.65	40.65	40.65	40.65
−0.5	−0.5	−0.5	0.5	5.83	5.38	5.83	7.54	40.75	40.75	40.75	40.8
−0.5	−0.5	0.5	−0.5	5.38	5.83	7.54	5.83	40.75	40.75	40.8	40.75
−0.5	0.5	−0.5	−0.5	5.83	7.54	5.83	5.38	40.75	40.8	40.75	40.75
0.5	−0.5	−0.5	−0.5	7.54	5.83	5.38	5.83	40.8	40.75	40.75	40.75
−0.5	−0.5	0.5	0.5	5.17	5.17	7.06	7.06	40.6	40.6	40.65	40.65
−0.5	0.5	0.5	−0.5	5.17	7.06	7.06	5.17	40.6	40.65	40.65	40.6
0.5	0.5	−0.5	−0.5	7.06	7.06	5.17	5.17	40.65	40.65	40.6	40.6
0.5	−0.5	−0.5	0.5	7.06	5.17	5.17	7.06	40.65	40.6	40.6	40.65
−0.5	0.5	−0.5	0.5	5.69	6.76	5.69	6.76	40.65	40.65	40.65	40.65
0.5	−0.5	0.5	−0.5	6.76	5.69	6.76	5.69	40.65	40.65	40.65	40.65
−0.5	0.5	0.5	0.5	5.12	6.42	6.84	6.42	40.5	40.5	40.55	40.5
0.5	0.5	0.5	−0.5	6.42	6.84	6.42	5.12	40.5	40.55	40.5	40.5
0.5	0.5	−0.5	0.5	6.84	6.42	5.12	6.42	40.55	40.5	40.5	40.5
0.5	−0.5	0.5	0.5	6.42	5.12	6.42	6.84	40.5	40.5	40.5	40.55

Table 2: Experimental data.

B Design symmetry

Due to the symmetry of the fan assembly, only six runs are actually needed to create a full 2^4 factorial design. For example, the run $-+-+ = (x_1 = -1, x_2 = +1, x_3 = -1, x_4 = +1)$ gives the same results as the run $+-+-$, when observing the response for the adjacent blade. Table 3 summarizes this symmetry for the experiments performed in this report.

This rotational symmetry of blade assemblies was observed by Tony O'Hagan in the 24 blade assembly data. In this study the symmetry can be observed as well in the response data in Table 2.

Representative run	Equivalent 2^4 runs
$----$	$----$
$---+$	$---+,--+-,-+--,+---$
$--++$	$--++,-+-+,+--+,+--+$
$-+-+$	$-+-+,+-+-$
$-+++$	$-+++,+-++,++-+,+++-$
$++++$	$++++$

Table 3: Rotational symmetry of the factorial 2^4 structure.