2.1 Linking Models and Reality:
Reification Strategies for a Rainfall-Runoff Model.

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* work done in collaboration with Michael Goldstein, Allan Seheult, Leanna House, (Dept. Mathematical Sciences, Durham University), Prof Keith Beven (Lancaster University) Funding: MUCM and EPSRC.

Overview

- Reification Strategies.
- The Rainfall-runoff model.
- Dynamic Reification.
- Modular Reification.

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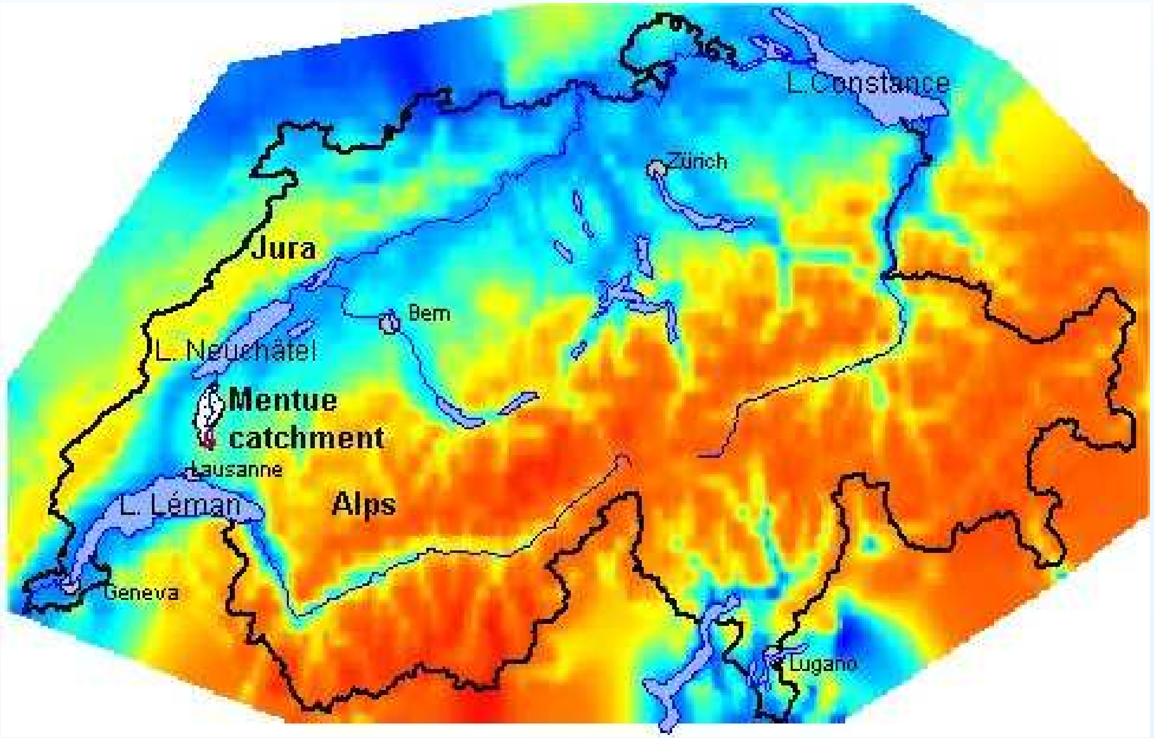
  - Dynamic Emulation and Reification.
  - Modular Emulation and Reification.
- The further down the list we go, the easier the Reification, but the harder the Emulation.

The Flood-Runoff Model of the Haute-Mentue catchment.

- West part of Switzerland, 15 km north of Lausanne, in the canton of Vaud.



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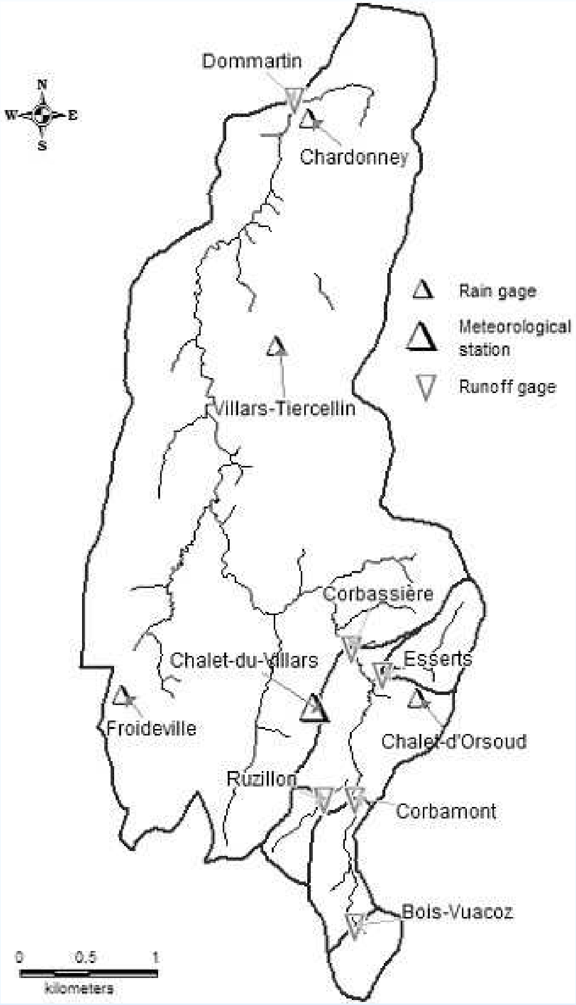

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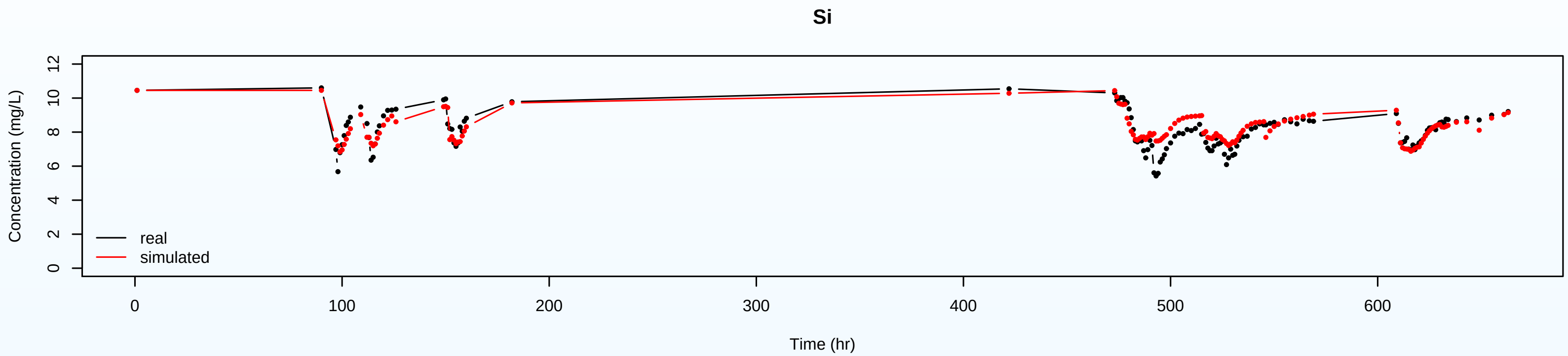
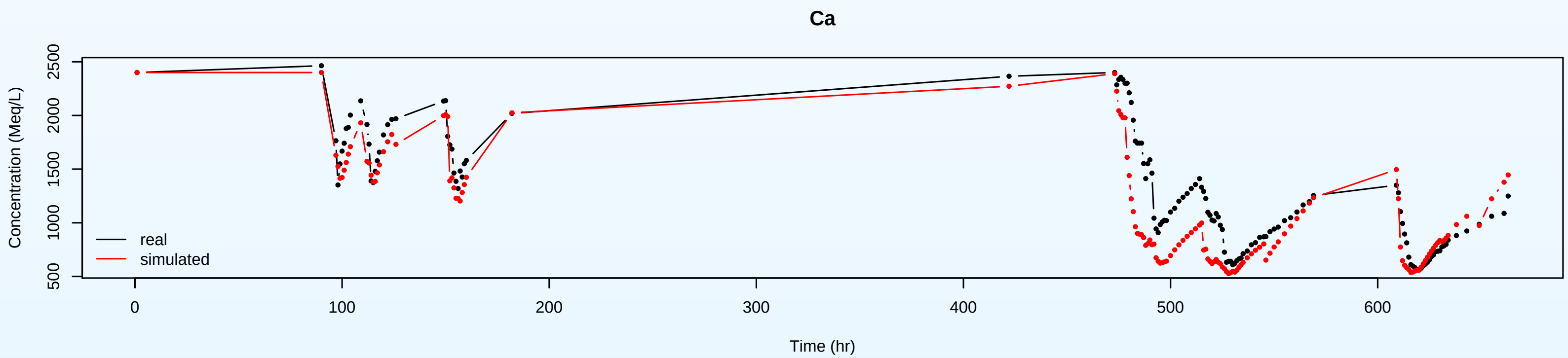
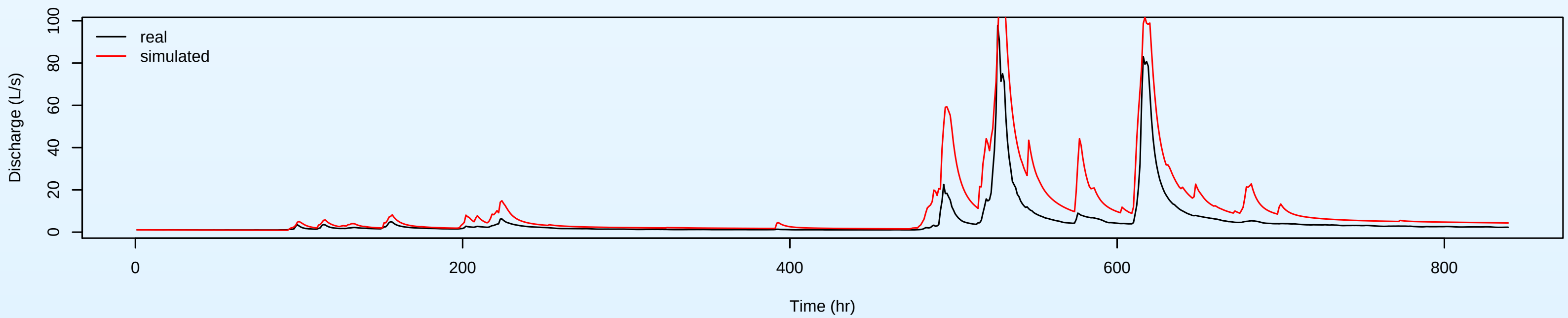
- Model takes measured rainfall rates $rain(t)$ and assumed evaporation rates $AET(t)$ as forcing functions.

Example of Model output







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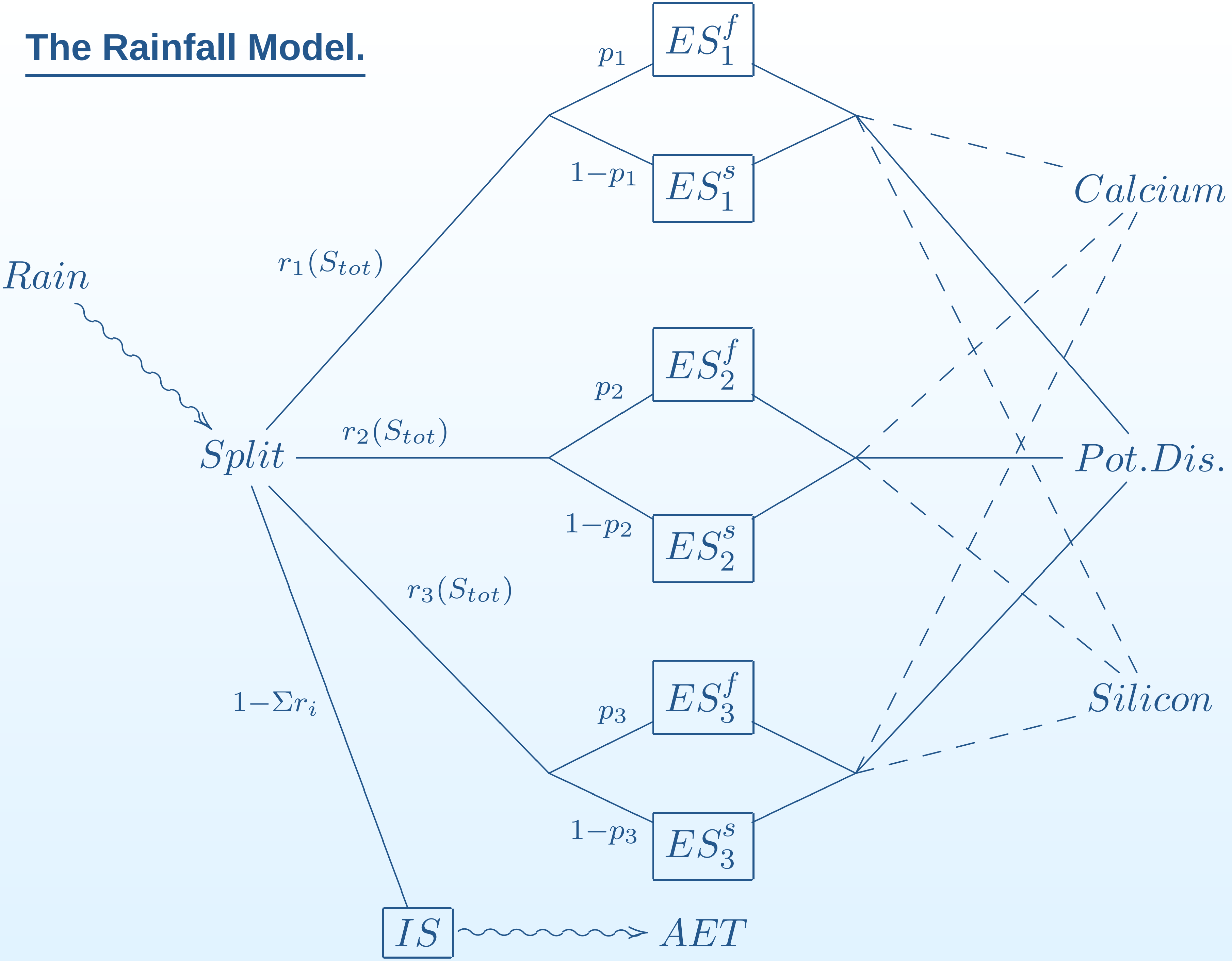

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- The model runs over 829 time steps, and produces outputs at each step.
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- Therefore we can emulate the 1-step simulator $f^{(1)}$ as it will be much simpler and this will allow us to track various uncertainties through time (e.g. forcing function uncertainty, model discrepancy).
- $f^{(1)}$ is a function that has 26 inputs: the 17 usual inputs to the model x, the forcing functions at time t $(rain(t), AET(t)) = A(t)$ and the 7 State Variables at time t-1: $(ES_i^{f/s}(t-1), S_{tot}(t-1)) = S(t-1)$.
- $f^{(1)}$ has 7 outputs that are the State Variables at time t: $(ES_i^{f/s}(t), S_{tot}(t)) = S(t)$.

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- The 7 state variable inputs S have such a dominant effect on the outputs of $f^{(1)}(x, A, S)$, it can be hard to pick up the more subtle impact of the other inputs.
- Emulation techniques such as active variables can be misleading.
- Accounting for other sources of uncertainty, as required in the Reification process, can still be difficult at this level (although is far easier than Direct Reification).

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- Easier for expert to consider improvements to an individual process, as opposed to the whole model.
- Downside: it is difficult to link the modular emulators together (but we already have this problem in the Dynamic Case).

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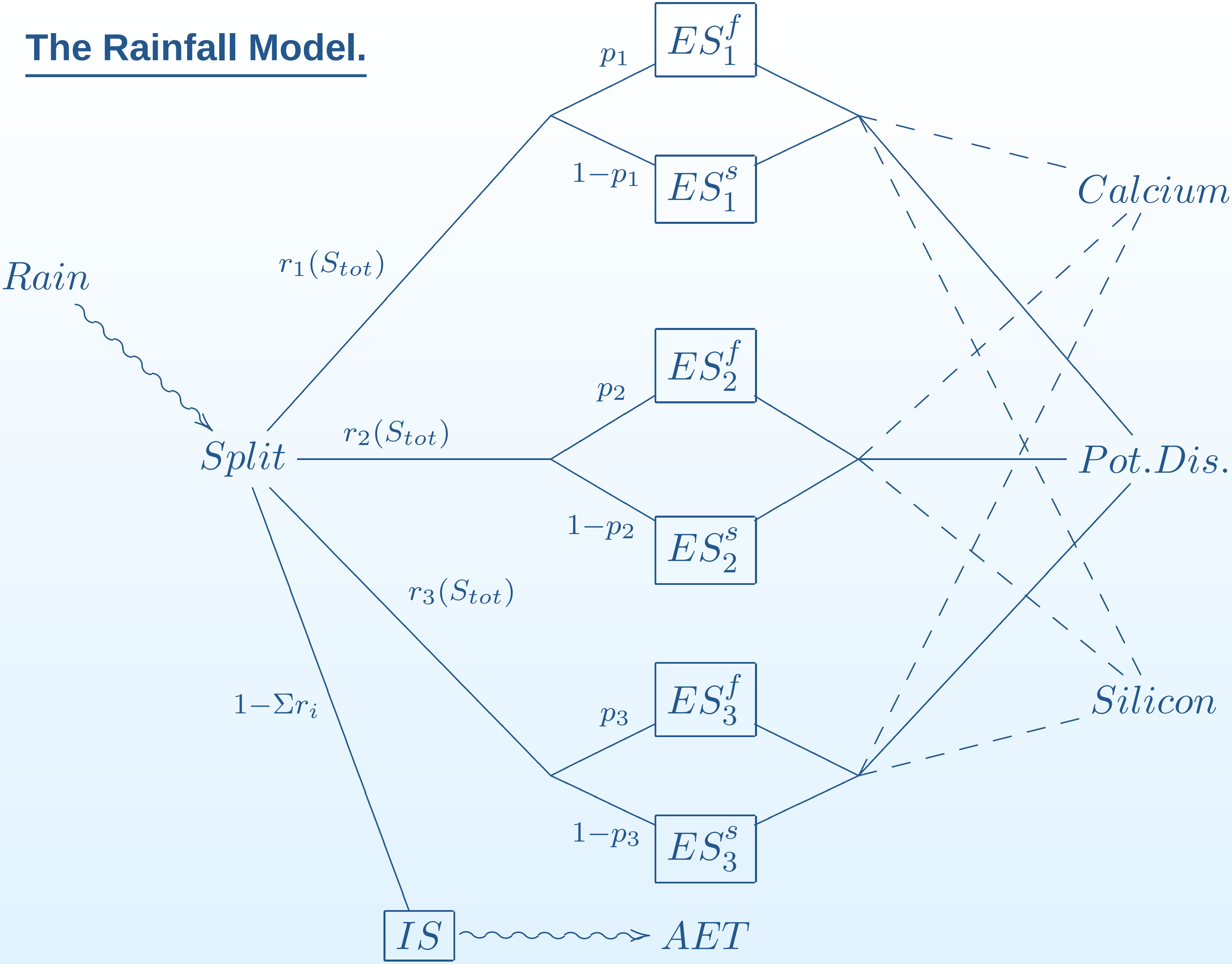
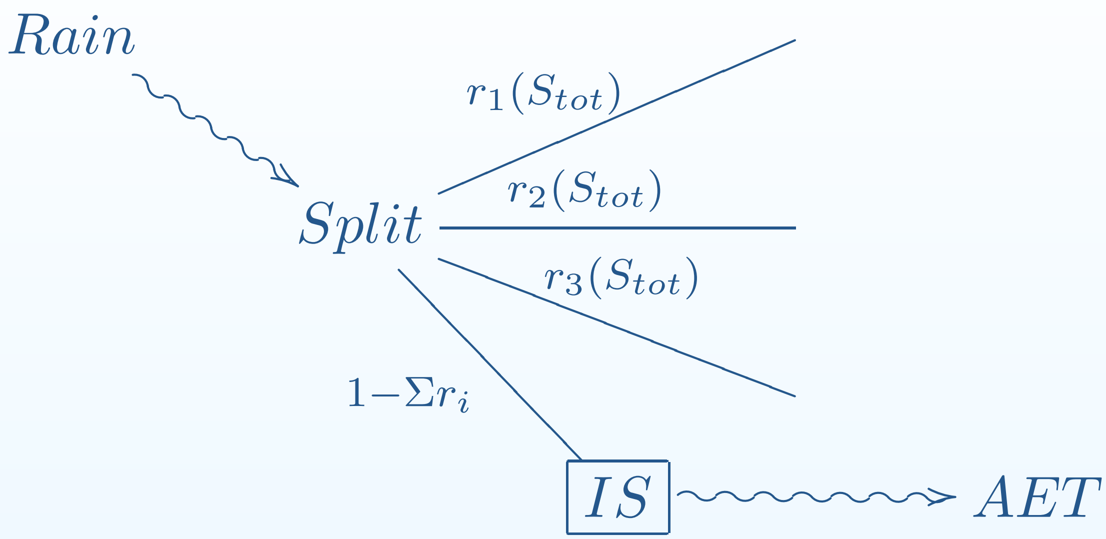
- Intend to improve emulators at later stage.

The Rainfall Model.



Module 1



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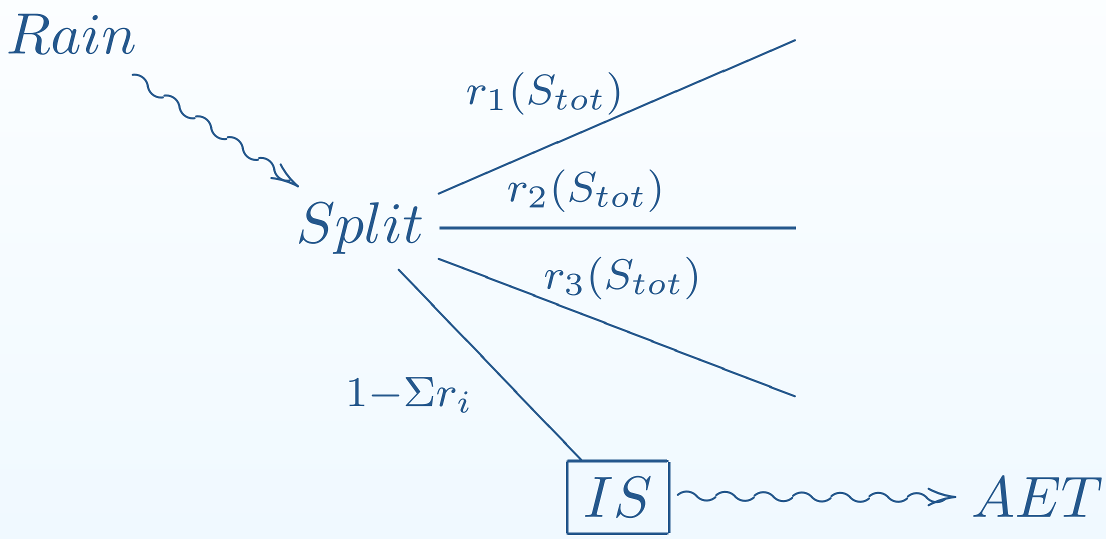
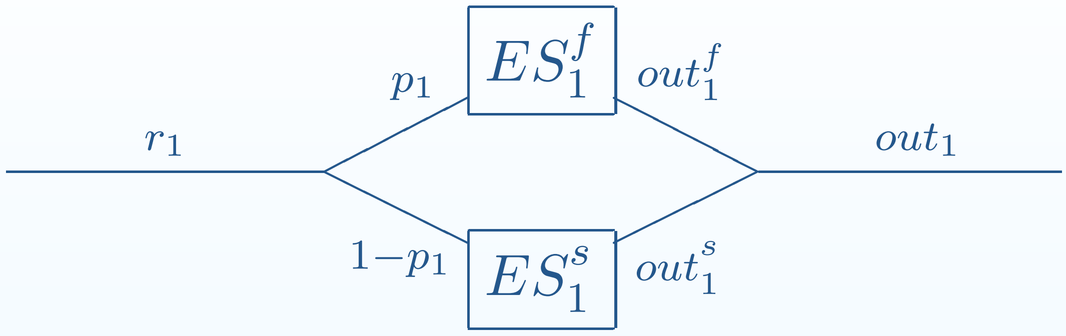


- Emulated the split function outputs r_i to reasonably high degree of accuracy.
- Find that the r_i depend on only $k_i, a_i, b_i, S_{tot}, rain$.
- Allowed incorporation of the uncertain forcing function rainfall.
- Will attempt to do internal reification on the r_i functions directly.

Module 2



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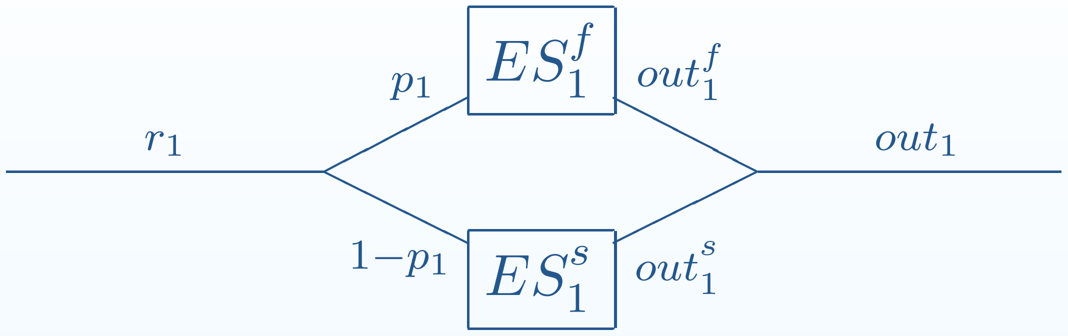
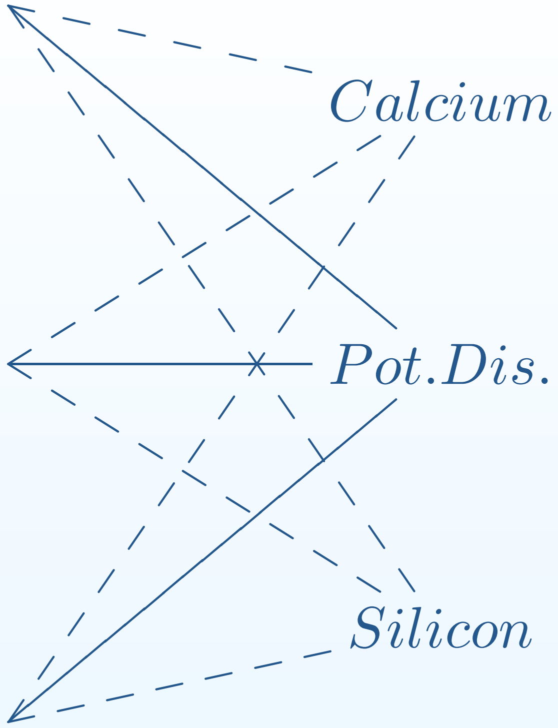


- Similarly we want to emulate and reify the above single ’compartment’
- We can choose how far ‘into the box’ we want to go.
- e.g. we find that $out_1^f(t)$ is a function that only depends on two inputs: $ES_1^f(t-1)$ and cf_1, and is trivial to emulate.
- Again can Reify this (or parts of this) compartment process.

Module 3



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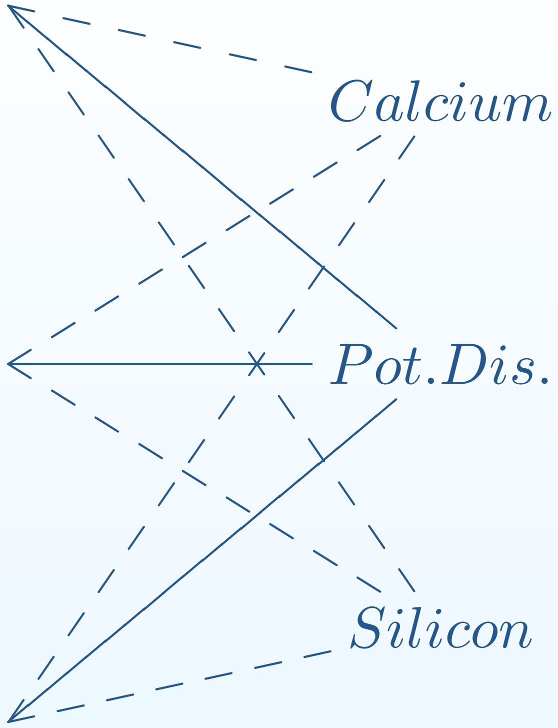

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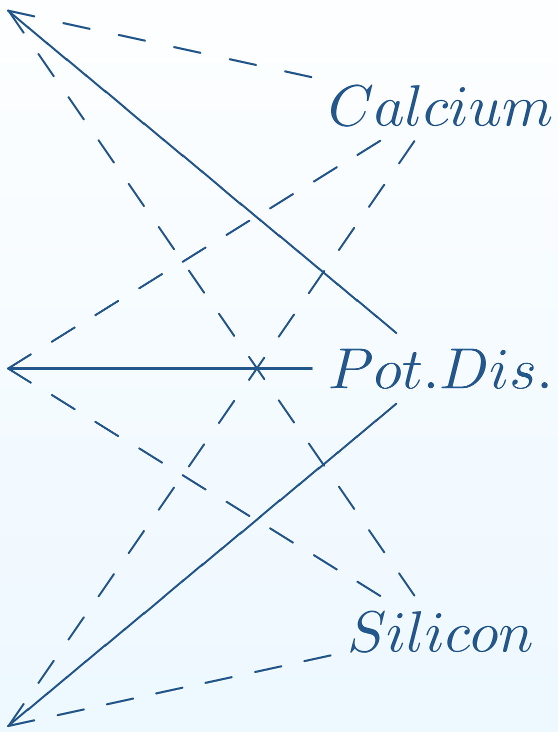

- Final outputs Calcium, Pot.Dis. and Silicon straightforward to emulate.
- Contains assumption of fixed Tracers that lead to Ca and Si concentration in rivers.
- Modular Emulation has allowed incorporation of uncertainty due to Tracers: first step in a Reification of this function.

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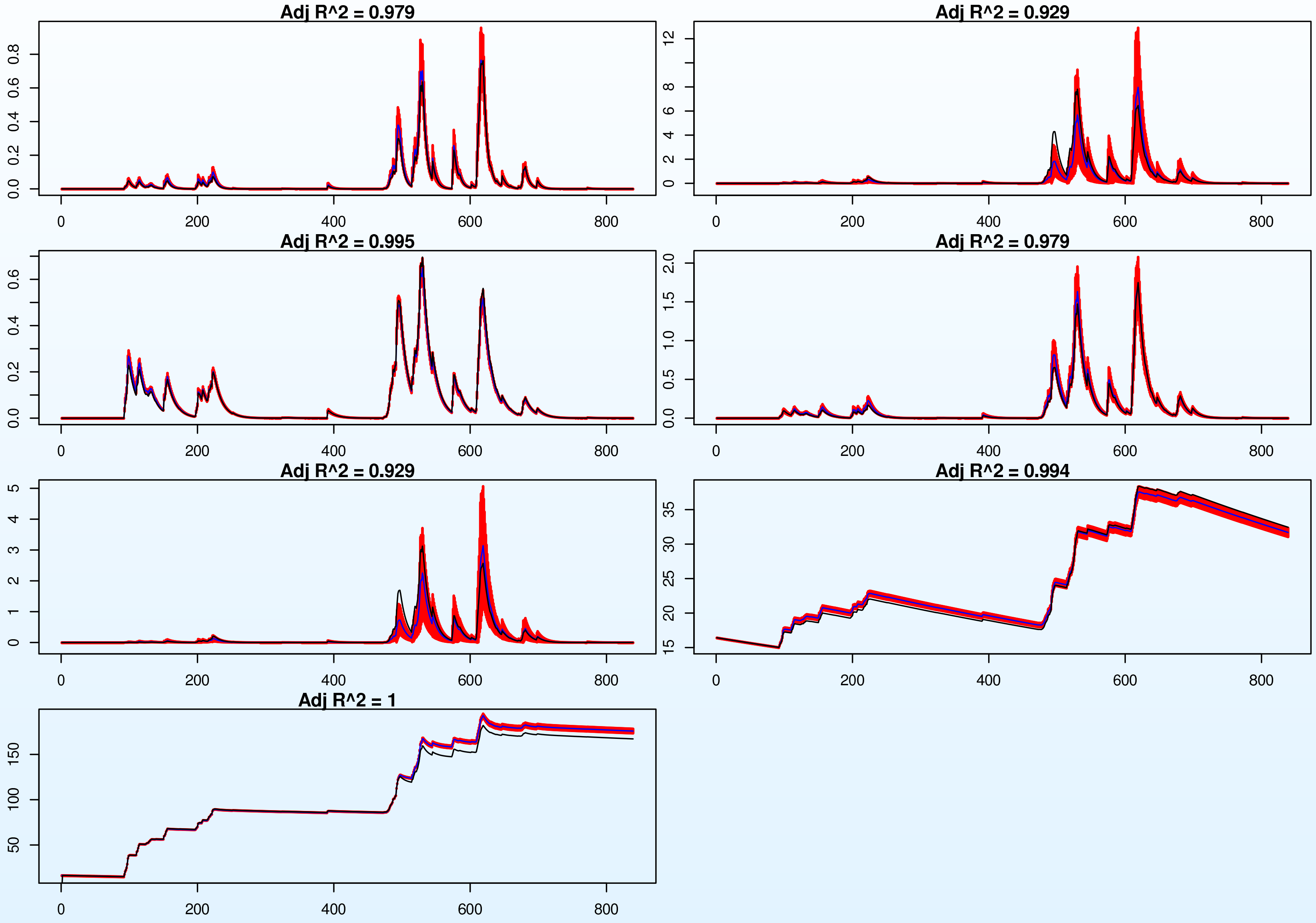

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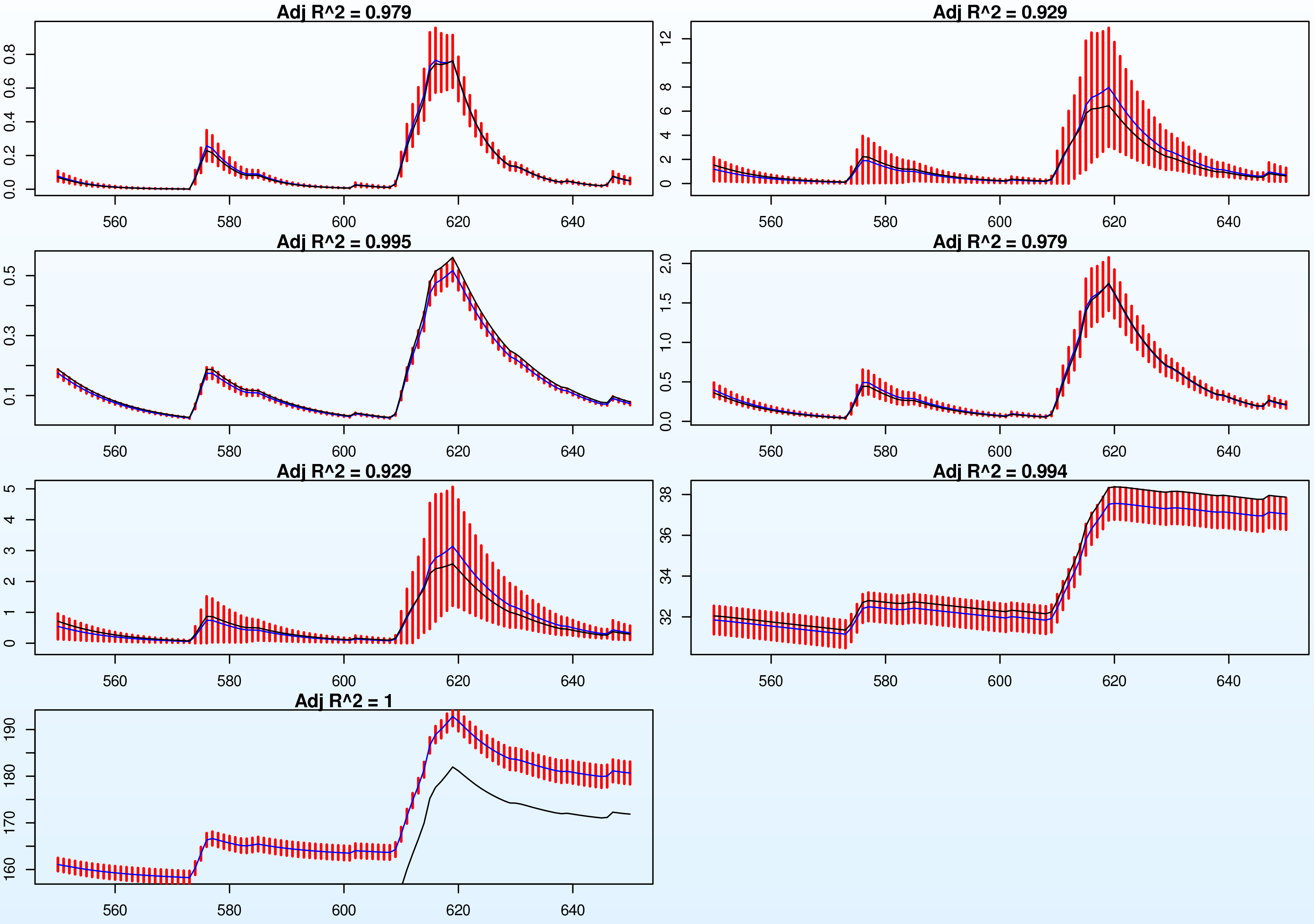

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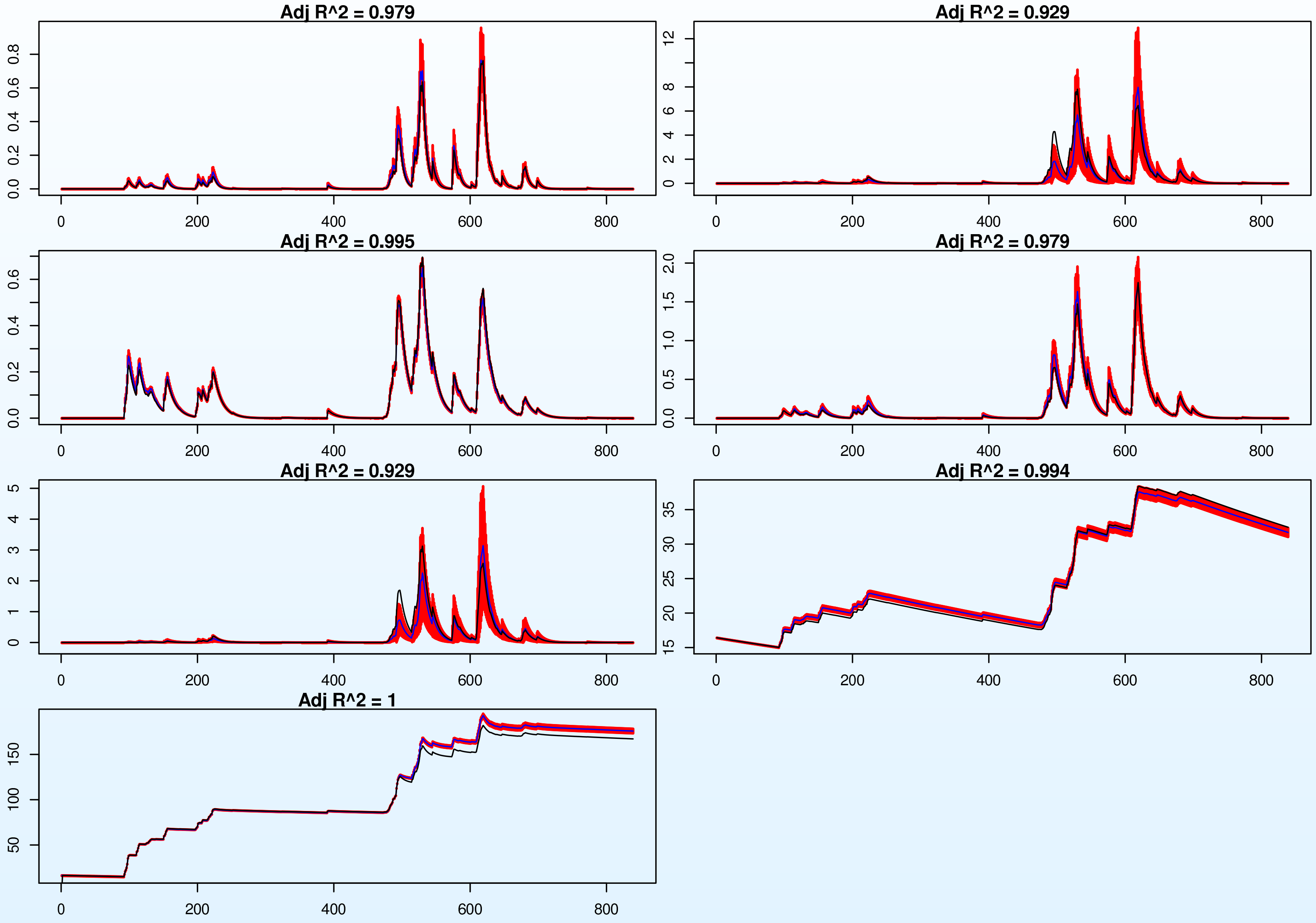

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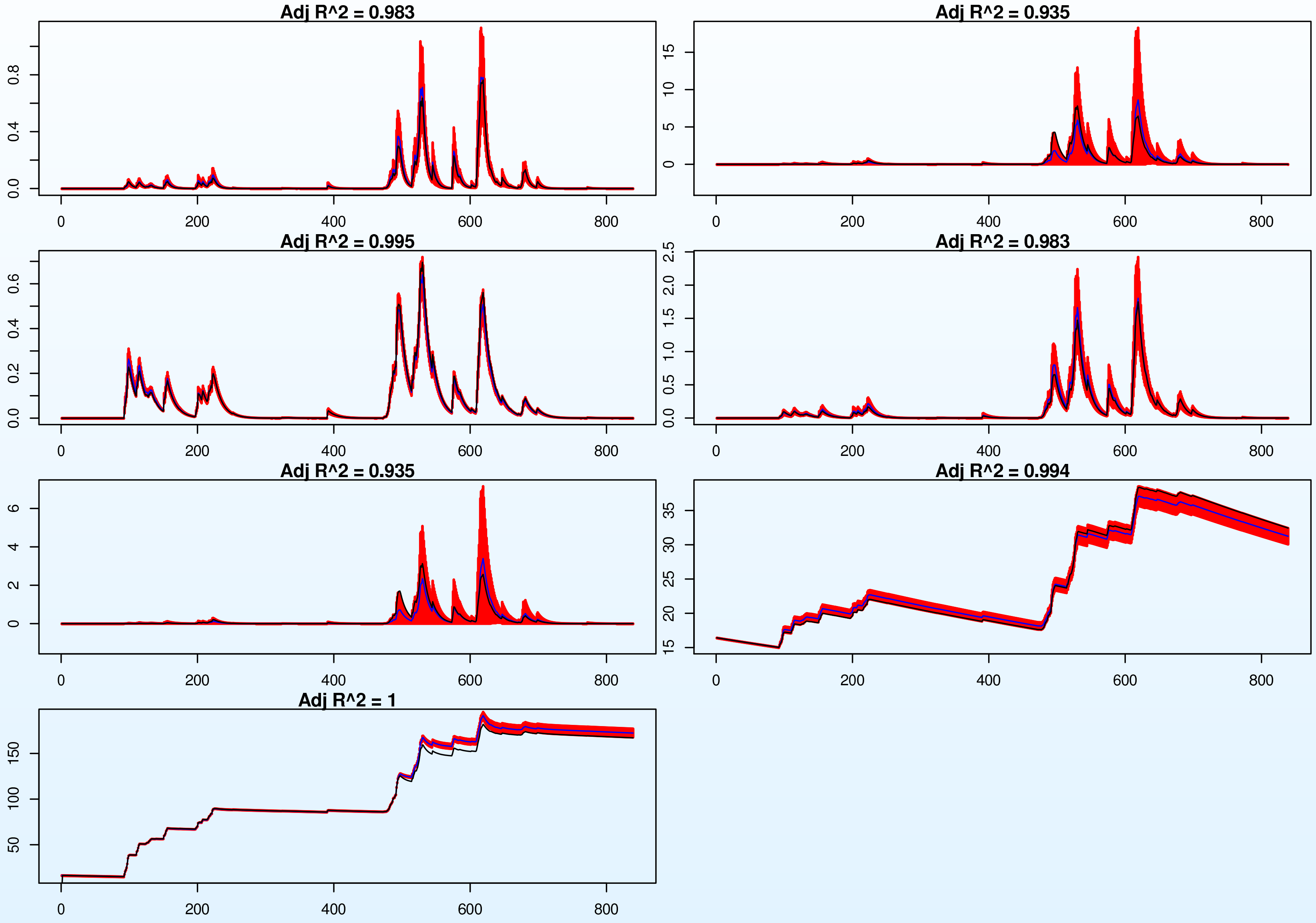

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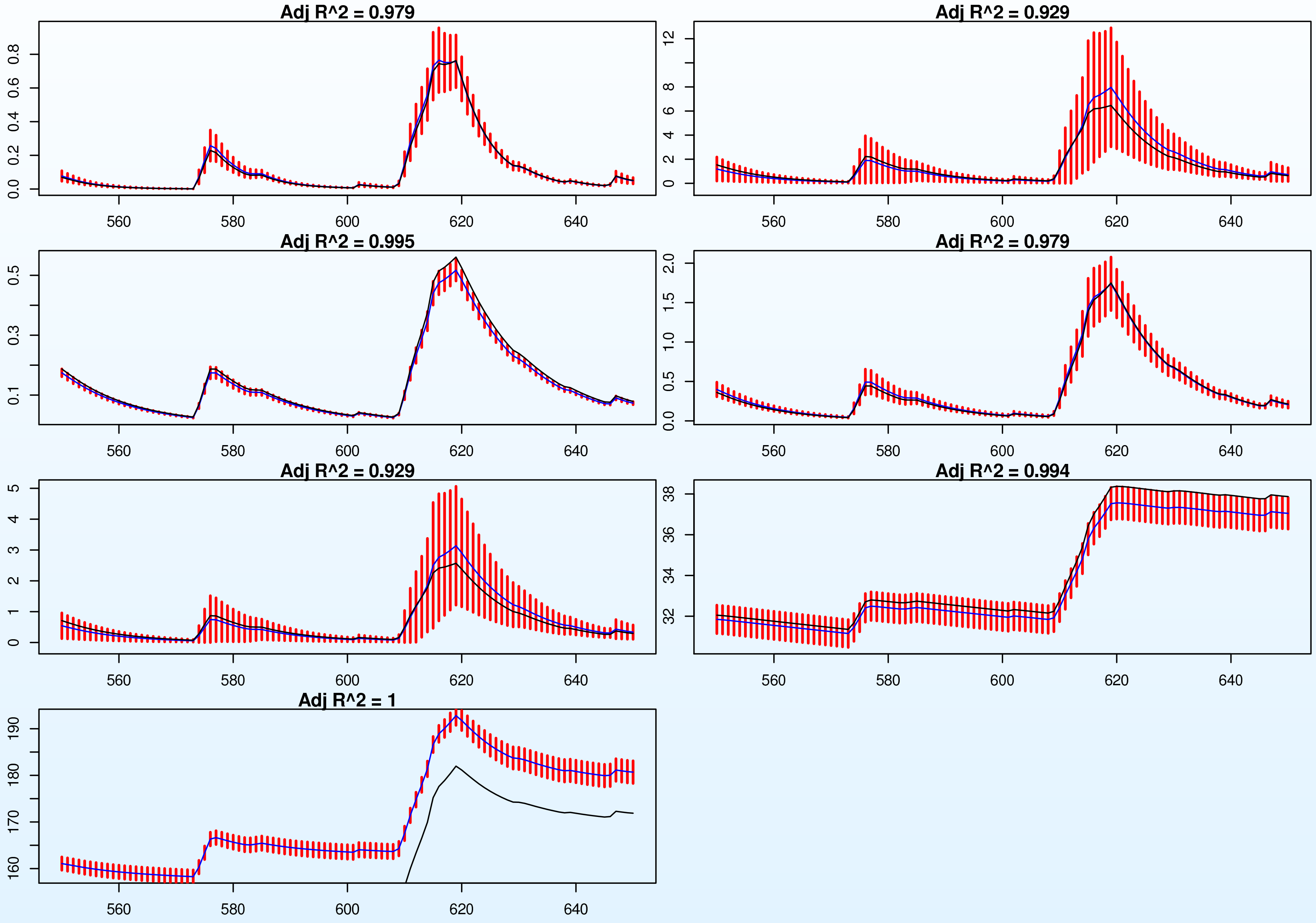

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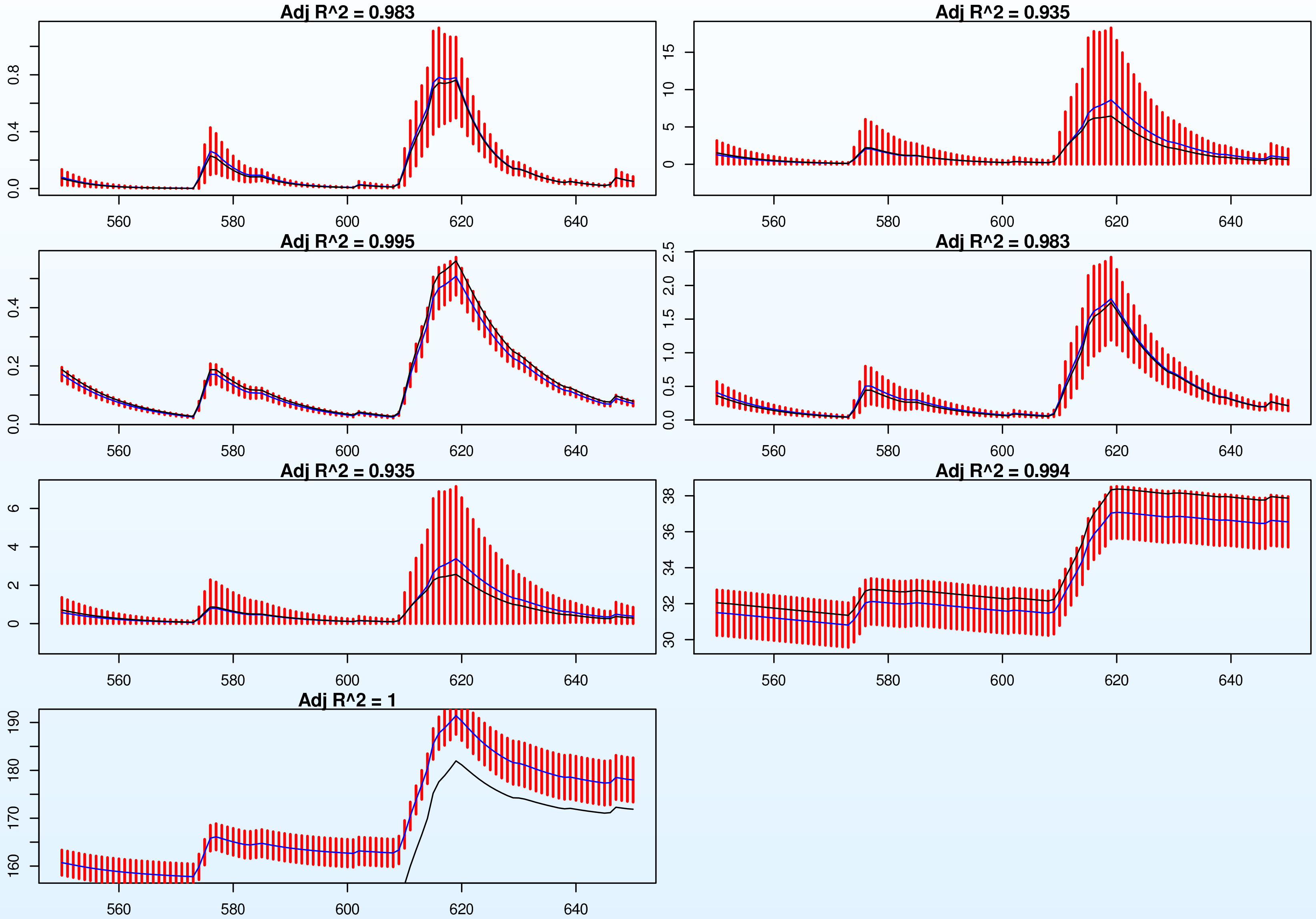

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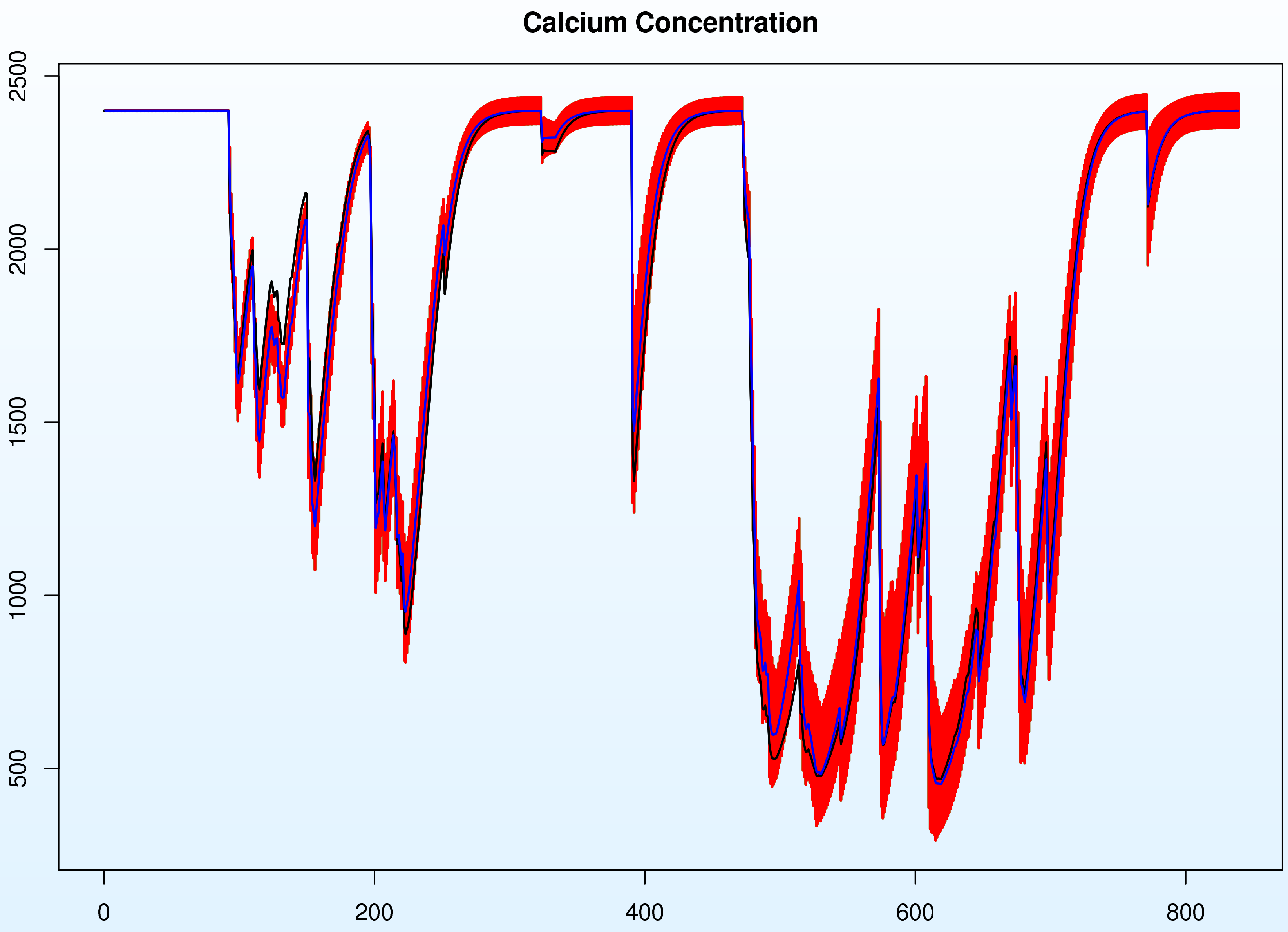

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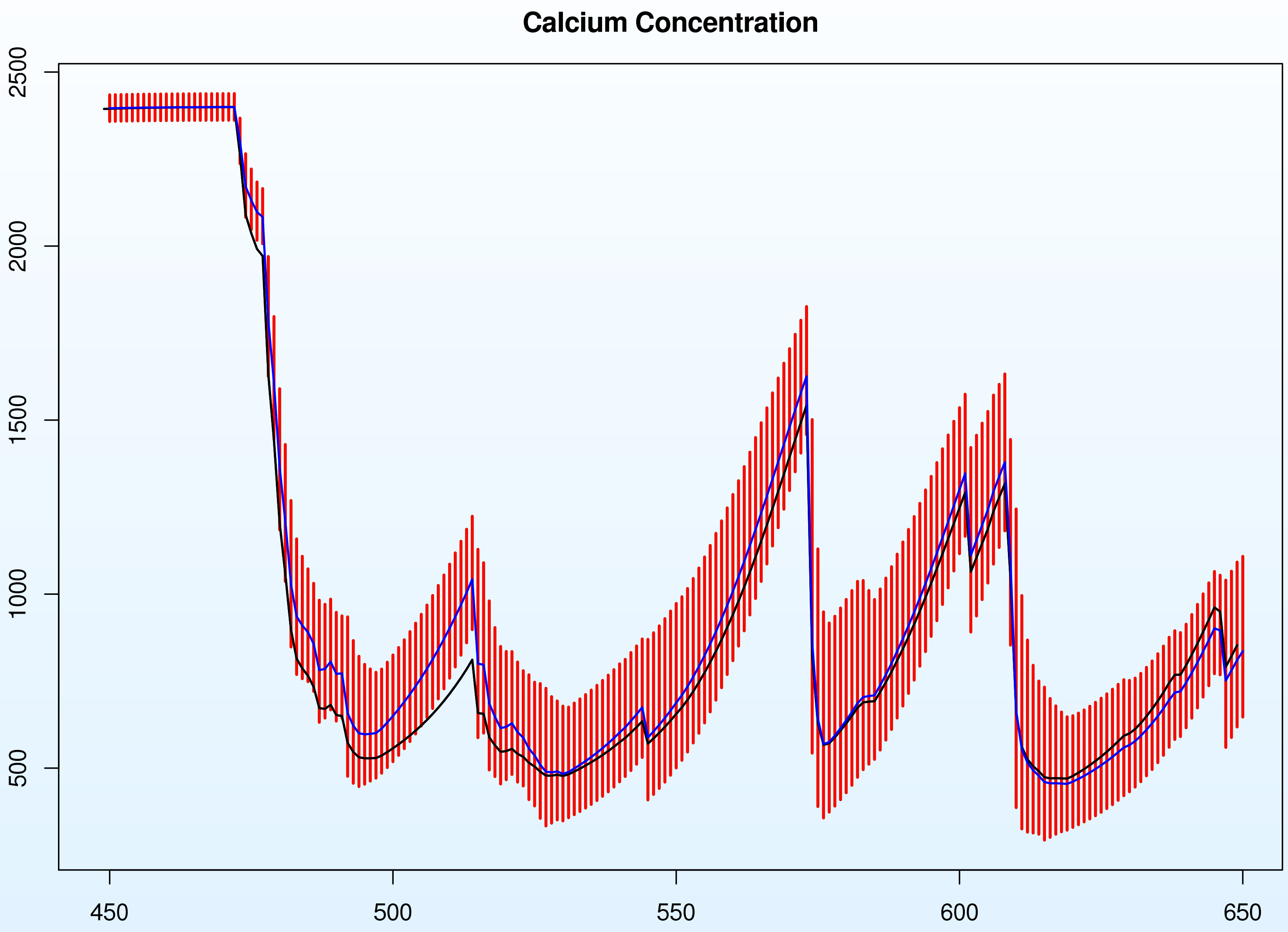
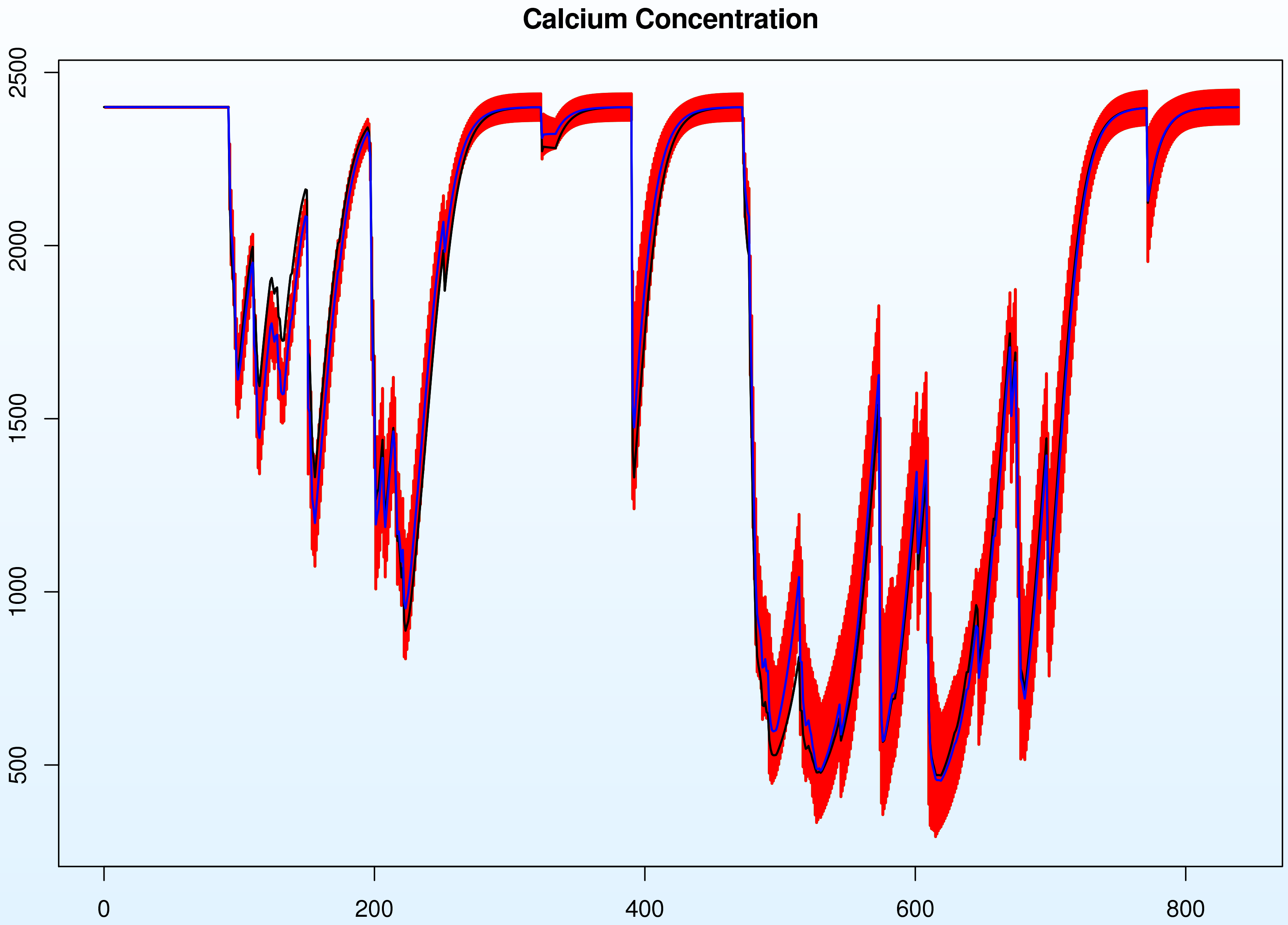


The Modular Emulator: Tracer Uncertainty



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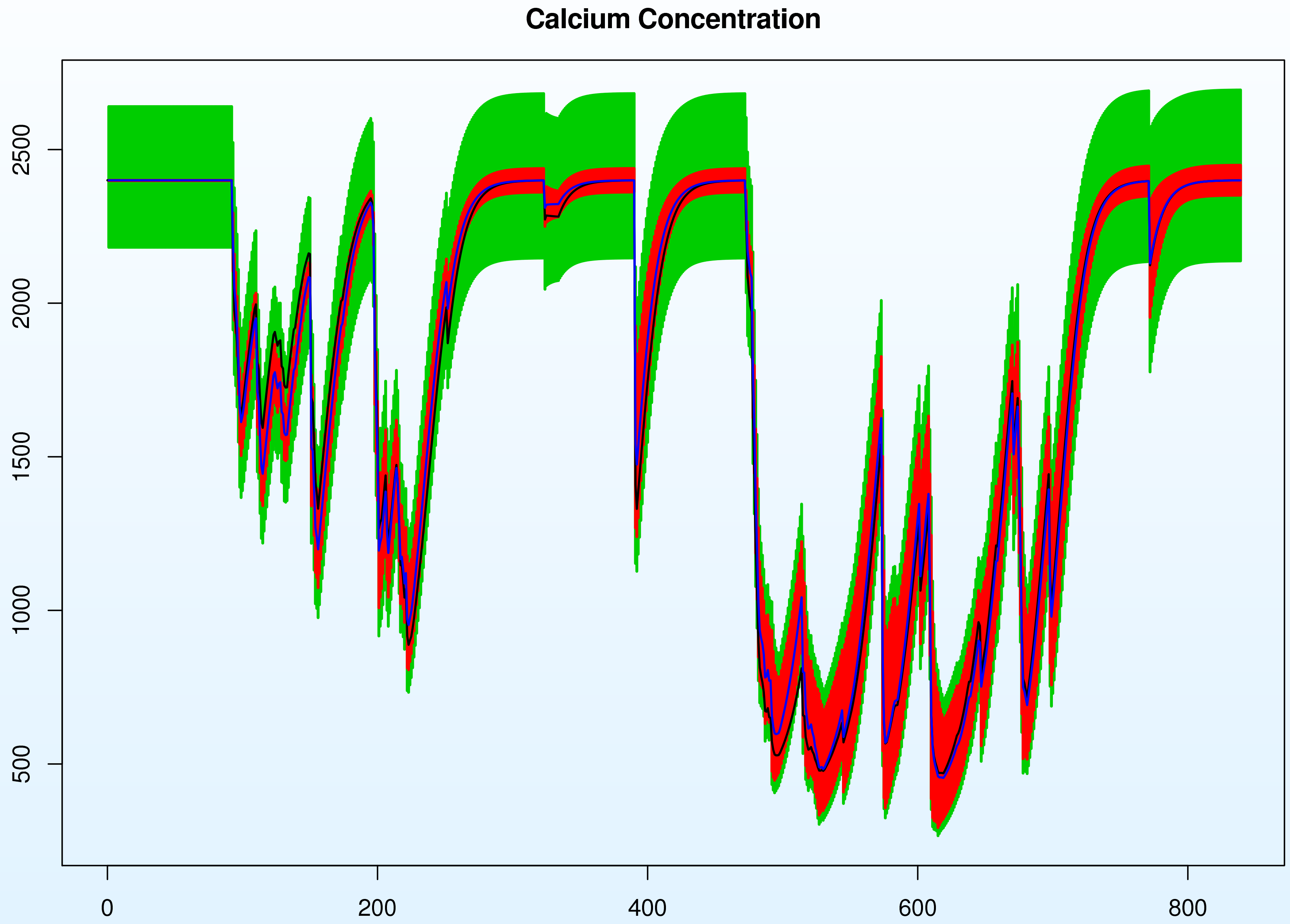

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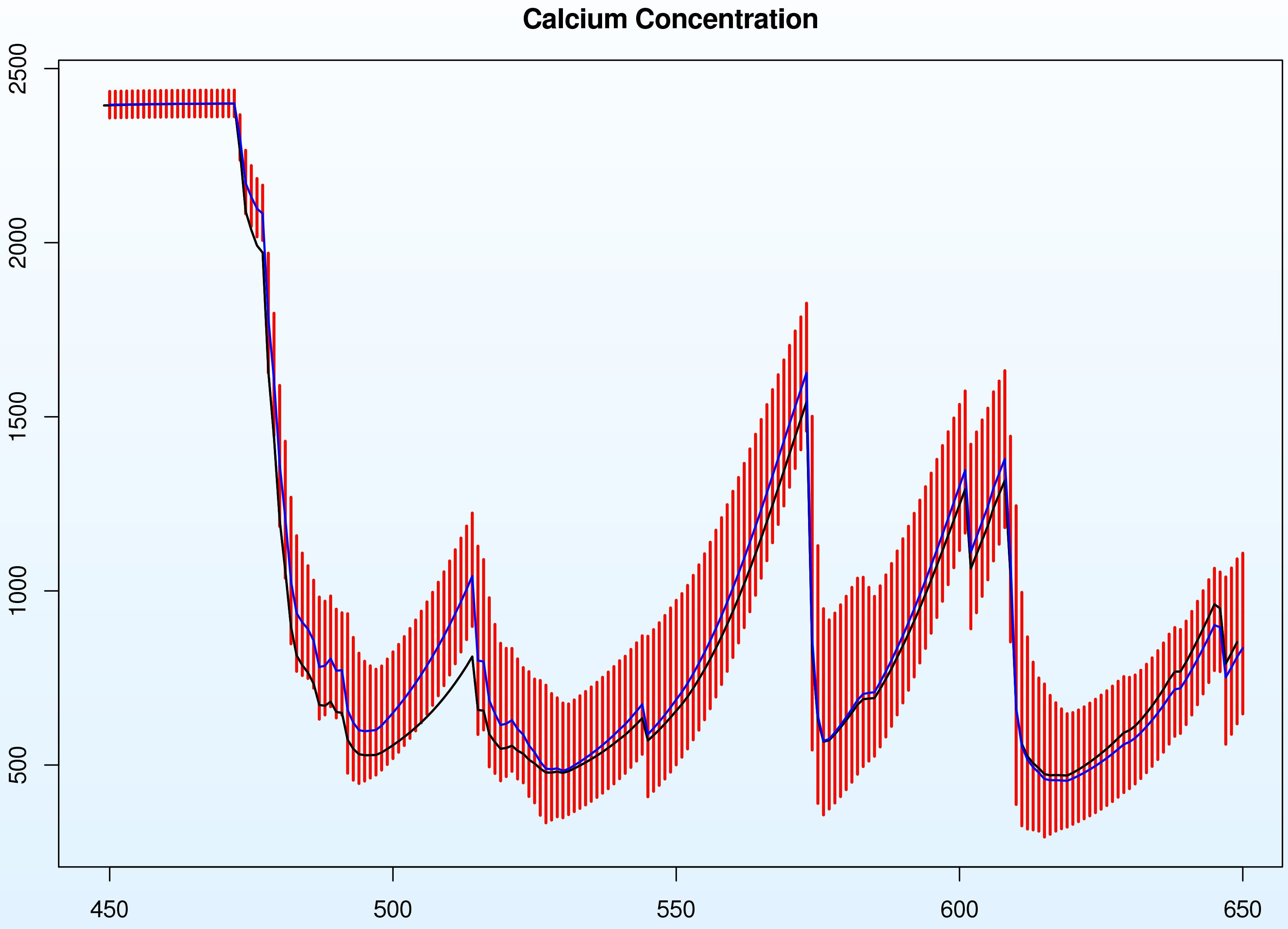

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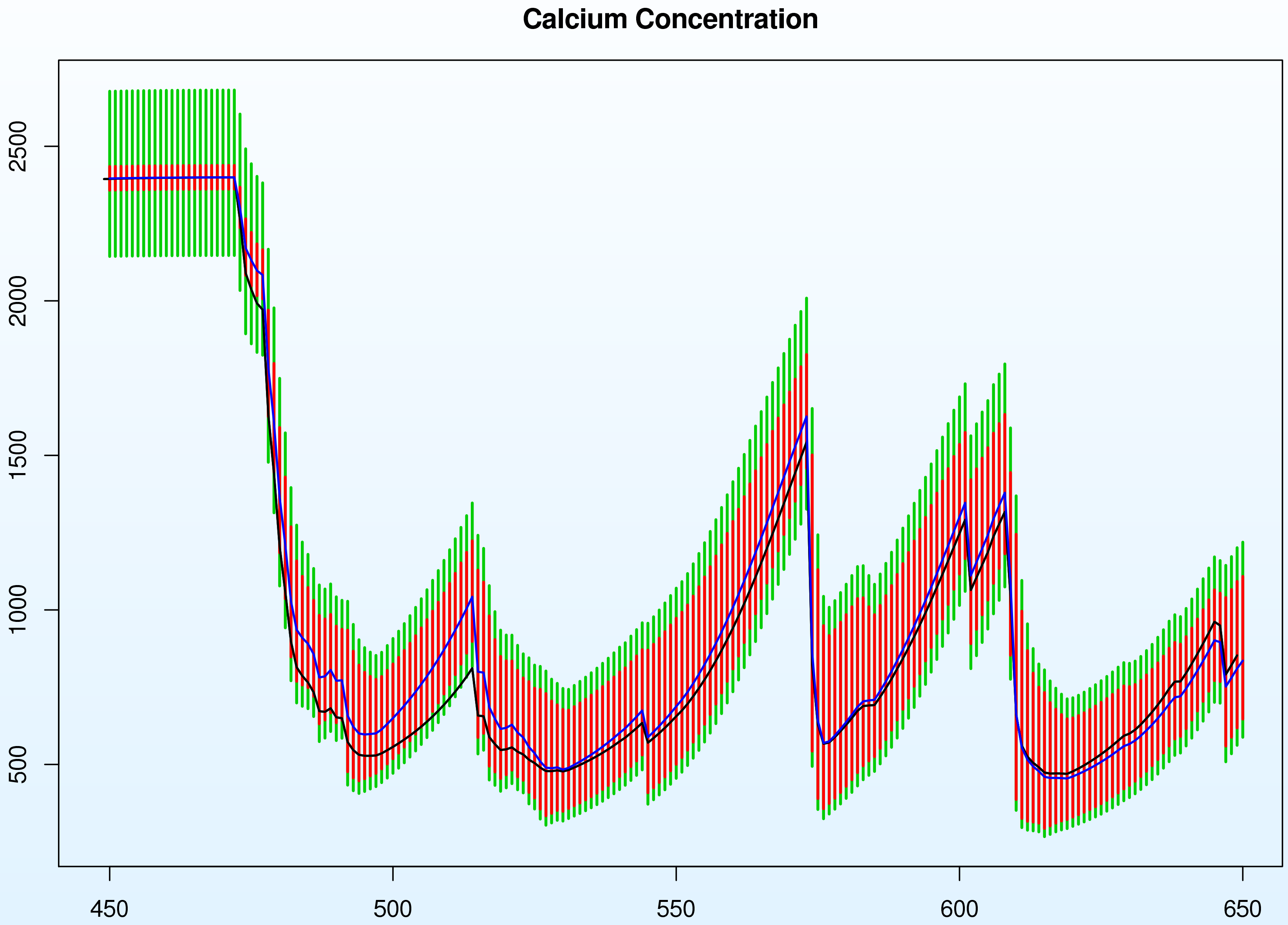

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- Lots more to say: Checking assumptions via simulation, comparisons of Direct Emulation with Dynamic and Modular Emulation, assessment of relative impact of different uncertainties, upgrading constant inputs to time dependent processes, External Reification...

References

M. Goldstein and J.C.Rougier. Probabilistic formulations for transferring inferences from mathematical models to physical systems (2005) SIAM journal on scientific computing, 26, 467-487.

M. Goldstein and J.C.Rougier. Reified Bayesian modelling and inference for physical systems (2006) In revision for JSPI.