Predicting avalanche characteristics by combining model evaluations and experimental results

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AVALANCHE!!

Avalanches: A Statistician's guide (on one slide)

- The characteristics of an avalanche, such as mass and velocity, depend on the topography, and on time-varying environmental variables such as
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- The general approach is to construct semi-empirical models, and then calibrate them to observations. But this requires a careful treatment of uncertainty, which is endemic.

The source of our experimental data

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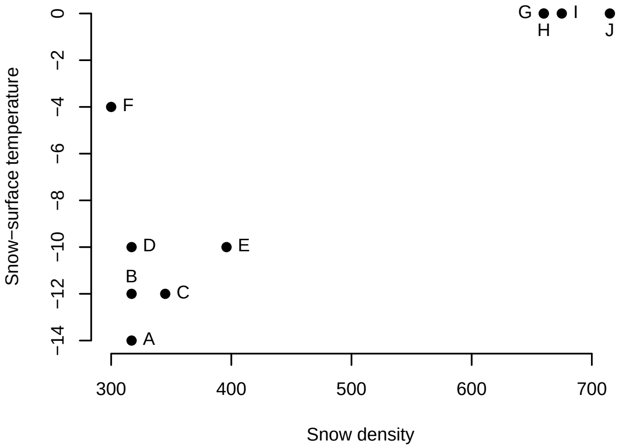
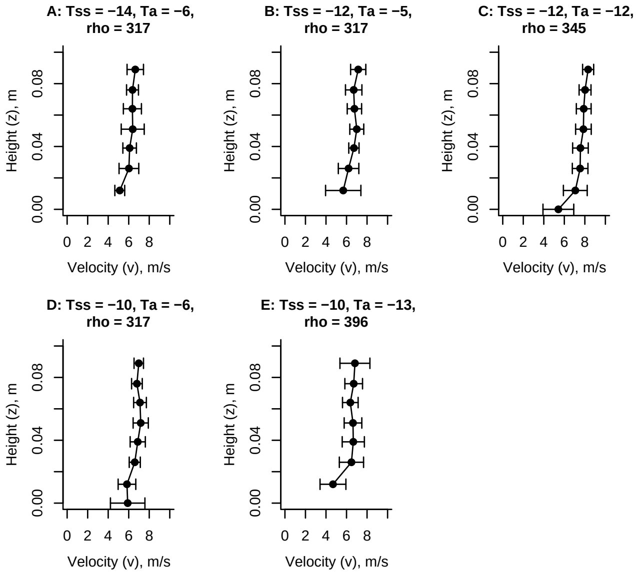
Environmental conditions for our experiments

Snow density (kg/m^3) and snow-surface temperature (°C) for ten large chute experiments.



The experimental data



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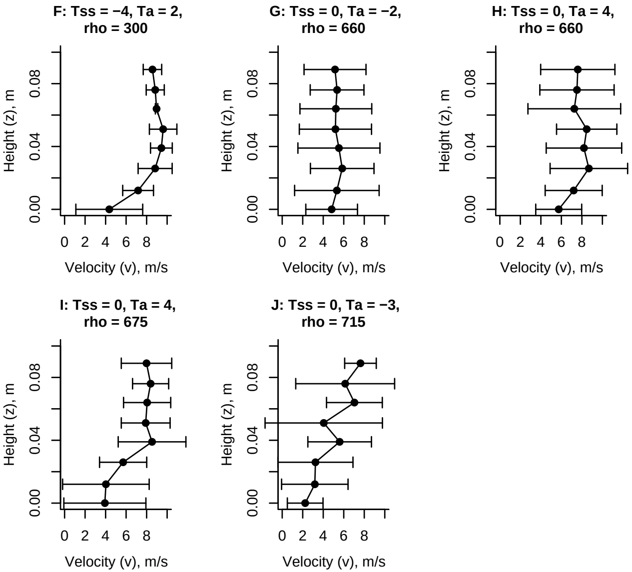
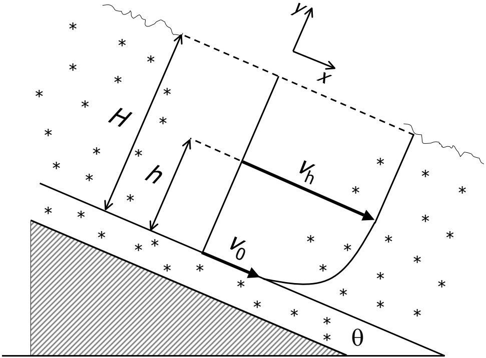


The Herschel-Bulkley model



- This is a two-layer model, with a shear-layer $[0, h)$, and a plug-layer $[h, H]$. It solves for the velocity profile $v(z)$.
- The model includes the effect of snow density ρ and snow-surface temperature T_{ss}, the environmental variables (or *treatments*).
- It contains uncertain parameters: $x = \left(v_0, \tau_c, \alpha, t_c\right)$.

Statistical model

1. Single treatment: $\mathbf{x}^*$ is the 'best' value of the uncertain parameters, $g(\cdot)$ is the HB model, and $\mathbf{v}, \mathbf{v}^{\text{obs}}$ the actual and observed velocity profiles

$$\mathbf{x}^* \xrightarrow{g} \mathbf{v} \longrightarrow \mathbf{v}^{\text{obs}}$$

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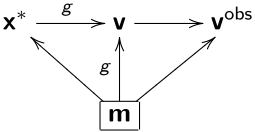

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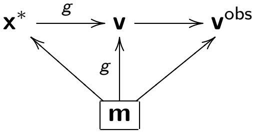
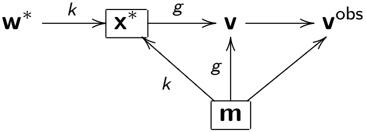


3. Reparameterised inputs, to isolate from treatment:



Sources of uncertainty

Summary: We treat z as an index variable; $\mathbf{m} = (\rho, T_{ss})$ as environmental variables; $\mathbf{x} = (v_0, \tau_c, \alpha, t_c)$ as uncertain model parameters; θ as fixed (32°), H as fixed (0.4m).

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   - Systematic in both height and treatment, increasing in height, independent of 'best' value of model parameters.
3. 'Best' value of model parameters, $\mathbf{x}^* = (v_0^*, \tau_c^*, \alpha^*, t_c^*)$
   - Our judgement of v_0^* depends on ρ, and τ_c^* on T_{ss}, in the general relation $\mathbf{x}^* = k(\mathbf{w}^*, m)$;
   - α^* and t_c^* are independent, and Gamma-distributed.

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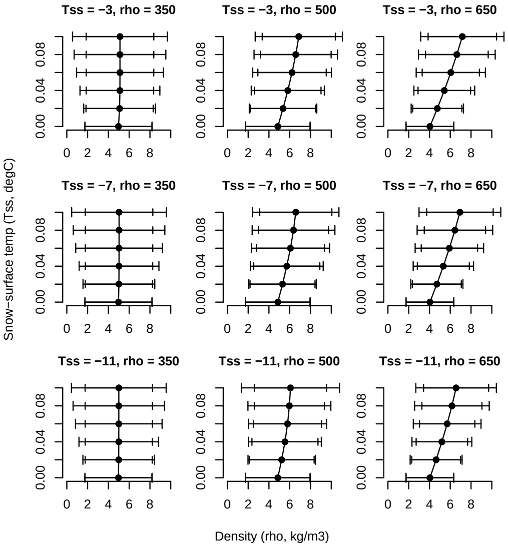
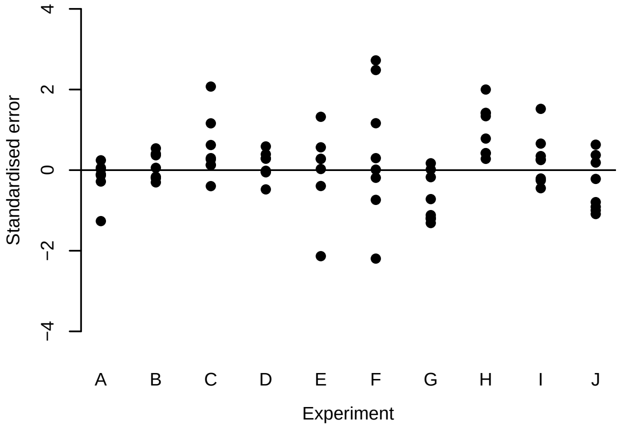
3. Data-based diagnostics, which compare our predictions of the observations with their actual values.

Prior-predictive validation of our judgements



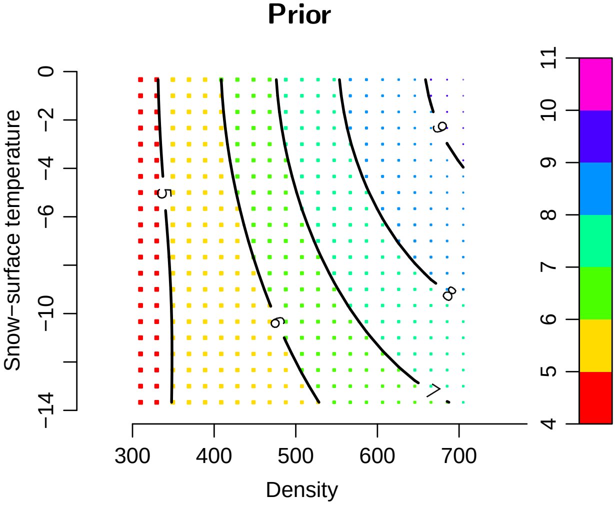
Data-based validation of our judgements

Leave-one-experiment-out diagnostics: prediction errors transformed so that they ought to have mean zero, standard deviation one, and be uncorrelated.



Effect of adjusting by observations

Plug-layer velocity, predicted mean and standard deviation, by treatment; std dev. on scale 0 m/s (full box) to 4 m/s (no box).



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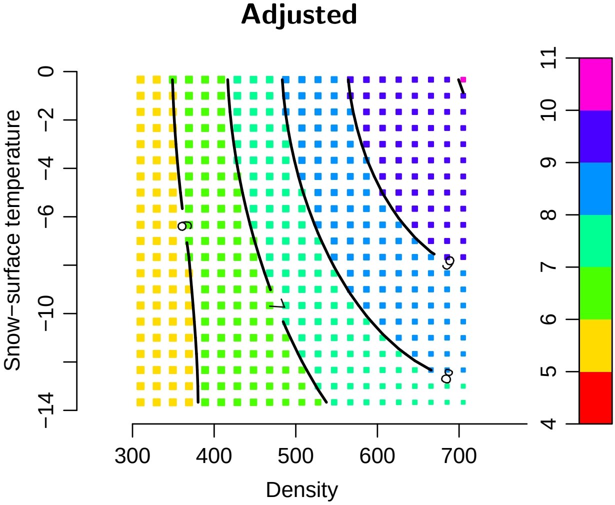

Summary

'Poor' physical model $+$ Noisy data $\Rightarrow$ Careful treatment of uncertainty.

- Judgements are an essential part of this process: they are the glue that join the model evaluations, the actual system behaviour, and the observations together.
- The 'engineering' approach, where the observational error is taken to dominate all other sources, fails to capture the important role of systematic error when the model is poor.
- Validation of our judgements happens through both predictive and data-based assessment.
- After adjusting by the observations, some of the predictive uncertainties may still be quite large: this is important to know.